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The RR charges of the A-type Gepner models

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Abstract: It is shown that the RR charges of Gepner models are not all accounted for by the usual tensor product and permutation branes. In order to characterise the missing D-branes we study the matrix factorisation approach to the description of D-branes for Gepner models. For each of the A-type models we identify a set of matrix factorisations whose charges generate the full lattice of quantised charges. The additional factorisations that are required correspond to generalised permutation branes.

Keywords: D-branes, Conformal Field Models in String Theory.

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1. Introduction

Most phenomenologically relevant string compactifications involve Calabi-Yau manifolds. From a conformal field theory point of view, string compactifications on a Calabi-Yau manifold can be described (at least at some specific points in their moduli space) in terms of Gepner models [1–4]. Gepner models are certain orbifolds of tensor products of minimal N=2 superconformal models. Since in most phenomenological string constructions D-branes play a crucial rôle, it is an important problem to construct and understand the D-branes for these models.

A certain class of D-branes for Gepner models can be relatively easily constructed: these are the so-called tensor product or Recknagel-Schomerus (RS) D-branes [5] that preserve the different N=2 superconformal algebras separately. A slight generalisation of this construction involves D-branes that preserve the different N=2 superconformal algebras up to a permutation, the so-called permutation branes [6]. It is then an interesting (and obvious) question to ask whether these different constructions account already for all the RR charges of these models. As we shall see, this is in general not the case. For example, of the 147 A-type Gepner models corresponding to six-dimensional Calabi-Yau manifolds there are 31 where the tensor product and permutation branes do not couple to all RR ground states. This analysis can be performed directly in conformal field theory.

Recently, a beautiful and simple characterisation of the (topological aspects of) B-type D-branes in N=2 theories has been proposed in unpublished work by Kontsevich. According to this idea one can characterise these D-branes in terms of matrix factorisations of the superpotential. The Kontsevich proposal has been supported in a number of papers [7 − 11] by analysing the possible B-type boundary terms for the Landau-Ginzburg action. For the case of the N=2 minimal models the Landau-Ginzburg results have also been shown to agree with the results obtained using conformal field theory methods [8, 12 − 16]. Furthermore, these techniques have been used to study D-branes on Calabi-Yau manifolds [17 − 19, 14, 16].

In this paper we want to use the matrix factorisation approach to understand which D-branes actually generate the RR charges for a certain class of Gepner models, the A-type models that correspond to the Fermat type Calabi-Yau manifolds. Unlike in conformal field theory where (so far) only tensor product and permutation branes are accessible, the matrix factorisations of a wide class of D-branes can be easily constructed and analysed. This approach therefore allows us to characterise the matrix factorisations that generate all the charges for these Gepner models.[1] We shall find that in addition to the factorisations that correspond to tensor product and permutation branes, only one new class of factorisations is required in order to account for the full quantised lattice of RR charges. By analogy with the identification of [14], we can identify the new factorisations with 'generalised permutation branes' that seem to exist whenever the (shifted) levels of the N=2 minimal models contain a non-trivial common factor. These D-branes are therefore very reminiscent of the generalised permutation branes of products of WZW models for which evidence was recently found in [20].

The paper is organised as follows. In section 2 we describe the RR states that carry the even cohomology charges for all A-type Gepner models. We then identify explicitly the Gepner models whose RR charges cannot be accounted for in terms of the usual (B-type) tensor product or permutation D-branes. In section 3, we briefly review the matrix factorisation approach to the analysis of B-type D-branes for Gepner models, and analyse a new class of rank 1 factorisations that appear whenever two exponents in the superpotential $W = \sum_i x_i^{h_i}$ contain a non-trivial common factor. We also determine the associated intersection matrices. In section 4 we use these results (as well as known formulae for the intersection matrices of tensor product and permutation factorisations) to identify, for all 147 A-type Gepner models, which matrix factorisations are required to account for all the RR charges. Our results are briefly summarised in section 5, and the complete list of the required matrix factorisations is given in the appendix.

[1] In this paper we shall only consider B-type D-branes that couple to the even cohomology charges. Using mirror symmetry, a similar analysis should be possible for the A-type D-branes.

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2. Ramond-Ramond ground states in Gepner models

Let us begin by setting up some notation for the description of the RR ground states in Gepner models [1]. A more comprehensive introduction to Gepner models can be found in [21]; our conventions are explained in more detail for example in [5, 22, 14].

2.1 Minimal models

Gepner models are orbifolds of tensor products of N=2 supersymmetric minimal models. The central charge of an N=2 minimal model is

$$c = \frac{3\,k}{k+2} \ , \tag{2.1}$$

where k is a positive integer. The representations of the bosonic subalgebra of the N=2 superconformal algebra are labelled by triples (l, m, s) of integers, where l takes the values $l = 0, \ldots, k$ and m and s are defined modulo $2k+4$ and 4, respectively. The three integers have to obey $l + m + s = 0 \mod 2$. Furthermore there is an identification

$$(l, m, s) \sim (k - l, m + k + 2, s + 2) \ . \tag{2.2}$$

The conformal weight h and the U(1)-charge q of states in the representation (l, m, s) are given by

$$h(l, m, s) = \frac{l(l+2) - m^2}{2k+4} + \frac{s^2}{8} \mod \mathbb{Z}, \qquad q(l, m, s) = \frac{s}{2} - \frac{m}{k+2} \mod 2\mathbb{Z}. \tag{2.3}$$

Representations with s even belong to the Neveu-Schwarz sector, while those with s odd belong to the Ramond sector.

Ramond ground states are characterised by the property that their conformal weight h obeys

$$h = \frac{c}{24} \ . \tag{2.4}$$

One can easily show that a sector $\mathcal{H}_{(l,m,s)}$ contains a Ramond ground state precisely if it is of the form $\mathcal{H}_{(l,l+1,1)}$ or $\mathcal{H}_{(l,-l-1,-1)}$. In this case, the formulae (2.3) are valid exactly (not only up to integers). Furthermore, we note that $(l, l+1, 1) \sim (l', -l'-1, -1)$ with $l' = k - l$.

2.2 Gepner models

In this paper we are interested in A-type Gepner models that describe six-dimensional Calabi-Yau compactifications. These are constructed starting with (at most) five N=2 minimal models with A-type modular invariants whose central charges add up to $c = 9$.[2] From a geometrical point of view these Gepner models correspond to Fermat type Calabi-Yau manifolds in $\mathbb{CP}^4$. There are precisely 147 such models that have been classified a long time ago [23–26].

[2] There exist another 21 Gepner models with $c = 9$ that involve more than five factors. As is explained in [3], these describe products of tori and K3s. We shall not consider them in this paper.

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In the Gepner model construction the tensor product of the minimal models is then projected onto states whose total U(1) charge is integral. The corresponding orbifold is in fact a simple current orbifold where the simple current J acts on the representation (l_i, m_i, s_i) of the i^{th} minimal model by

$$J : (l_i, m_i, s_i) \longrightarrow (l_i, m_i + 2, s_i) \ . \tag{2.5}$$

This is to say that in the n^{th} twisted sector the right-movers are shifted by J^n relative to the left-movers. The order H of the orbifold group is given by the least common multiple of the shifted levels, $H = \text{lcm}\{k_i + 2\}$.
The Ramond-Ramond sectors that appear in the orbifold theory are thus of the form

$$\bigotimes_{i=1}^{5} \mathcal{H}_{(l_i, m_i+n, s_i)} \otimes \bar{\mathcal{H}}_{(l_i, m_i-n, \bar{s}_i)} \ , \tag{2.6}$$

where $n = 0, 1, \ldots, H-1$ denotes the twisted sector, and $s_i, \bar{s}_i$ take values $-1, 1$. Because of the orbifold projection the labels m_i are subject to the integrality condition

$$\sum_{i=1}^{5} \frac{m_i}{k_i + 2} \in \mathbb{Z} \ . \tag{2.7}$$

[Recall that $\sum_{i=1}^{5} \frac{1}{k_i+2} = 1$.] The labels s_i and $\bar{s}_i$ in (2.6) are restricted in a way that depends on the GSO-projection. The GSO-projection that is compatible with B-type RR states is

$$\sum_{i=1}^{5} \left(\frac{s_i}{2} + \frac{\bar{s}_i}{2} \right) \in 2\mathbb{Z} \ . \tag{2.8}$$

In fact, it is easy to see that all RR sectors (2.6) that satisfy the integrality condition (2.7) as well as the B-type condition $q_L + q_R = 0$ on the left-/right-moving U(1)-charge necessarily obey (2.8).

The RR ground states of a Gepner model are then those states that are RR ground states in each minimal model factor.

2.3 RR-charges of D-branes in Gepner models

The RR charges we are interested in correspond to the even-dimensional cohomology of the Calabi-Yau manifold, whose charges are carried by the B-type D-branes. These D-branes can only couple to sectors whose total left-moving U(1)-charge q_L is opposite to its right-moving charge, $q_L = -q_R$. A special class of such branes are the tensor product branes [5] which preserve the individual N=2 superconformal symmetries of the five minimal models. They can therefore only couple to sectors with opposite U(1)-charges in each factor, $q_L^{(i)} = -q_R^{(i)}$. Another class of branes whose conformal field theory description is known are the permutation branes [6] which can occur if two or more levels are equal. In the simplest case of two equal levels k_1 and k_2, the permutation brane can only couple to sectors satisfying $q_L^{(1)} = -q_R^{(2)}$ and $q_L^{(2)} = -q_R^{(1)}$. If there are three or more equal levels, more permutations than just single transpositions are possible, but as will become apparent below, these will not be relevant for our considerations.

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In this section we want to analyse whether these two types of branes already account for all the RR charges of the above Gepner models. As is clear from the above discussion, a tensor product brane can only couple to the RR ground states in the sectors

$$\bigotimes \mathcal{H}_{(l_i,l_i+1,1)} \otimes \bar{\mathcal{H}}_{(l_i,-l_i-1,-1)} \;, \tag{2.9}$$

where $m_i = l_i + 1$ have to obey the integrality condition (2.7). Generically, the states in (2.9) will come from the n^{th} twisted sector where n satisfies $n = l_i + 1 \mod k_i + 2$. In the special situation when k_i is even and $l_i = \frac{k_i}{2}$, also $n = 0 \mod k_i + 2$ is allowed. For some Gepner models, all RR ground states are of this form, and thus all RR charges can be generated by tensor product branes.

There are however a number of examples for which some of the RR charges can only be accounted for in terms of permutation branes. (The simplest example is the theory with $(k = 7)^3(k = 1)^2$, as was already mentioned in [14].) A permutation brane in two factors (for ease of notation we are ignoring here the remaining three factors) couples to RR ground states in the sectors

$$\big(\mathcal{H}_{(l,l+1,1)} \otimes \bar{\mathcal{H}}_{(l,-l-1,-1)}\big) \otimes \big(\mathcal{H}_{(l,l+1,1)} \otimes \bar{\mathcal{H}}_{(l,-l-1,-1)}\big) \;. \tag{2.10}$$

It is easy to see that a permutation brane involving more than two factors does not couple to any new RR ground states; such branes only couple to RR ground states that can be built from the ground states in (2.10) and (2.9).

It is natural to ask whether there are RR ground states to which neither tensor product nor permutation branes can couple. As we have mentioned above, the B-type condition requires that the total U(1) charges satisfy $q_L = -q_R$. A RR ground state in a sector (2.6) has to have the same value for l_i in the left-moving and right-moving part, so it satisfies $q_L^{(i)} = \pm q_R^{(i)}$. In a given sector we now order the factors such that the first r factors satisfy $q_L^{(i)} = q_R^{(i)}$ and the last $(5 - r)$ factors satisfy $q_L^{(i)} = -q_R^{(i)}$. There is an ambiguity whenever some of the $q_L^{(i)}$ are zero. This allows us to always choose r to be even. Namely, assume that r is odd and $q_L^{(i)} \neq 0$ for all i. Then necessarily $s_i = \bar{s}_i$ for $i = 1, \ldots, r$ and $s_j = -\bar{s}_j$ for $j = r + 1, \ldots, 5$. This contradicts the GSO condition (2.8). So if r is odd there is at least one factor with $q_L^{(i)} = 0$ and we can lower or raise r to an even value.

The tensor product case corresponds to the case $r = 0$, so we are only interested in $r = 2, 4$. The case $r = 4$ actually never occurs which can be seen as follows: From $q_L = -q_R$ and $q_L^{(5)} = -q_R^{(5)}$ it follows immediately that

$$q_L - q_L^{(5)} = -(q_R - q_R^{(5)}) \;. \tag{2.11}$$

Since $q_L^{(i)} = q_R^{(i)}$ for $i = 1, \ldots, 4$ we also have $q_L - q_L^{(5)} = +(q_R - q_R^{(5)})$, and thus $q_L = q_L^{(5)}$. The integrality condition (2.7) implies that q_L is a half-integer. On the other hand $q_L^{(5)}$ is of the form

$$q_L^{(5)} = \frac{1}{2} - \frac{l_5 + 1}{k_5 + 2} \tag{2.12}$$

which is never half-integer. The case $r = 4$ can thus be excluded.

This leaves us with the case $r = 2$. The charges of the individual factors then satisfy

$$q_L^{(1)} = q_R^{(1)} \qquad\qquad q_L^{(2)} = q_R^{(2)} \qquad\qquad q_L^{(i)} = -q_R^{(i)}\ , \quad i = 3,4,5\ . \tag{2.13}$$

Together with the B-type condition $q_L = -q_R$ this implies that

$$q_L^{(1)} + q_R^{(2)} = q_L^{(2)} + q_R^{(1)} = 0\ . \tag{2.14}$$

Without loss of generality we may assume that the first left-moving Ramond ground state is in the sector $\mathcal{H}_{(l_1,l_1+1,1)}$, while the second is in the sector $\mathcal{H}_{(l_2,-l_2-1,-1)}$. To achieve (2.14) we thus need

$$\frac{l_1+1}{k_1+2} = \frac{l_2+1}{k_2+2}\ . \tag{2.15}$$

This condition can only be satisfied if the shifted levels k_1+2 and k_2+2 have a non-trivial common divisor d,

$$d = \gcd(k_1+2, k_2+2)\ . \tag{2.16}$$

For $d = 2$, however, the only solution to (2.15) is $l_i = k_i/2$ which implies $q_{L/R}^{(i)} = 0$. Then we are back to the tensor product case, so we can assume $d > 2$. If the levels are equal, $k_1 = k_2$, we recover the RR ground states that couple to permutation branes. New RR ground states can only be expected if $d > 2$ and $k_1 \neq k_2$. These RR ground states can only exist in sectors whose twist n is trivial in the first two factors, so

$$n = 0 \mod r_1 r_2 d\ ,$$

where $k_i + 2 = r_i d$. The procedure to find all of these states is now as follows. In each Gepner model we look for levels which are different and have a non-trivial common divisor $d > 2$. Then we investigate the twisted sectors with a twist $n \in r_1 r_2 d \cdot \mathbb{Z}$ and see whether there exists a RR ground state satisfying (2.15) and the integrality condition (2.7).

The analysis can be implemented on a computer. Out of the 147 Gepner models of A-type, there are 31 models with RR ground states of type $r = 2$ which do not couple to tensor product or permutation branes — these have been collected in table 1. In the simplest example the levels are $(6,6,4,2,1)$. The relevant two levels are $k_3 = 4$ and $k_5 = 1$ whose shifted levels have a common divisor $d = 3$. There are RR ground states of the type $r = 2$ in the 6^{th} and 18^{th} twisted sectors. For example, for $n = 6$ the relevant ground state appears in the sector

$$\begin{array}{lccccc} \text{L} & (1,-2,-1)_6 \otimes & (1,-2,-1)_6 \otimes & (1,2,1)_4 \otimes & (1,2,1)_2 \otimes & (0,-1,-1)_1 \\ \text{R} & (1,2,1)_6 \otimes & (1,2,1)_6 \otimes & (1,2,1)_4 \otimes & (1,-2,-1)_2 \otimes & (0,-1,-1)_1\ , \end{array}$$

where the first line describes the left- and the second line the right-moving representations, and the indices denote the levels k_i. One easily checks that this combination of representations appears in the $n = 6$ sector (i.e. is of the form (2.6) with $n = 6$), and that it satisfies the integrality condition (2.7). On the other hand, it is clear that this RR ground state cannot couple to any tensor product or permutation brane.

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Levels	(i,j)	n
(6, 6, 4, 2, 1)	(3, 5)	(6, 18)
(8, 4, 3, 3, 1)	(2, 5)	(6, 12, 18, 24)
(13, 8, 2, 2, 1)	(1, 5)	(15, 45)
(16, 7, 2, 2, 1)	(2, 5)	(9, 27)
(18, 4, 3, 2, 1)	(2, 5)	(6, 18, 42, 54)
(18, 10, 3, 1, 1)	(1, 3)	(20, 40)
(18, 18, 13, 1, 0)	(3, 4)	(15, 45)
(26, 19, 2, 1, 1)	(1, 3)	(28, 56)
(26, 19, 10, 1, 0)	(2, 4)	(21, 63)
(28, 8, 3, 1, 1)	(2, 3)	(10, 20)
(28, 18, 2, 1, 1)	(2, 3)	(20, 40)
(33, 13, 12, 1, 0)	(2, 4)	(15, 45, 75, 135, 165, 195)
(34, 34, 7, 1, 0)	(3, 4)	(9, 27)
(43, 28, 7, 1, 0)	(3, 4)	(9, 27, 63, 81)
(43, 34, 3, 2, 0)	(1, 3)	(45, 135)
(46, 14, 2, 1, 1)	(2, 3)	(16, 32)
(52, 25, 7, 1, 0)	(3, 4)	(9, 45)
(58, 13, 10, 1, 0)	(2, 4)	(15, 45)
(70, 22, 7, 1, 0)	(3, 4)	(9, 27, 45, 63)
(86, 31, 6, 1, 0)	(2, 4)	(33, 99, 165, 231)
(89, 76, 5, 1, 0)	(1, 3)	(91, 455)
(97, 20, 7, 1, 0)	(3, 4)	(9, 27, 45, 63, 81, 117, 135, 153, 171, 189)
(98, 23, 3, 2, 0)	(2, 3)	(25, 75)
(108, 13, 9, 1, 0)	(2, 4)	(15, 45, 75, 105, 135, 195, 225, 255, 285, 315)
(124, 19, 7, 1, 0)	(2, 4)	(21, 105)
	(3, 4)	(9, 27, 45, 81, 99, 117)
(178, 18, 7, 1, 0)	(3, 4)	(9, 27, 45, 63, 81, 99, 117, 135, 153, 171)
(214, 25, 6, 1, 0)	(2, 4)	(27, 81, 135, 189)
(236, 49, 5, 1, 0)	(2, 4)	(51, 153, 255, 459, 561, 663)
(292, 47, 5, 1, 0)	(2, 3)	(49, 245)
(340, 17, 7, 1, 0)	(3, 4)	(9, 27, 45, 63, 81, 99, 117, 135, 153, 189, 207, 225, 243, 261, 279, 297, 315, 333)
(628, 43, 5, 1, 0)	(2, 4)	(45, 135, 225, 405, 495, 585)

Table 1: The Gepner models with RR ground states of type $r = 2$ in the factors i, j. n denotes the sectors in which they appear.

Our analysis thus shows that in general the tensor product or permutation branes do not account for all the charges. Furthermore, it suggests that the only additional D-brane construction that is required to account for these charges involves two factors whose shifted levels have a non-trivial common factor. This is very reminiscent of the generalised permutation branes for factors of SU(2) WZW models for which evidence was recently found in [20].

In the following sections we shall analyse the same problem using the matrix factorisation point of view. We shall be able to reproduce the above results, but we shall also be able to identify the matrix factorisations that will actually account for all the RR charges. In particular, we shall find that the new factorisations that are required are indeed a natural generalisation of the factorisations that correspond to permutation branes.

3. Matrix factorisations

D-branes in Gepner models can also be analysed in terms of orbifolds of Landau-Ginzburg models that flow in the IR to the relevant superconformal field theory. In particular, B-type D-branes in Landau-Ginzburg models can be described in terms of matrix factorisations. This approach goes back to unpublished work of Kontsevich, and the physics interpretation of it was given in [7−9, 12, 10, 11]; for a good review of this material see for example [27].

3.1 Generalities

According to Kontsevich's proposal, D-branes in Landau-Ginzburg models correspond to matrix factorisations of the superpotential W,

$$E\,J = J\,E = W \cdot \mathbf{1} \ , \tag{3.1}$$

where E and J are $r \times r$ matrices. This condition can be more succinctly written as

$$Q^2 = W \cdot \mathbf{1} \ , \qquad \text{where} \qquad Q = \begin{pmatrix} 0 & J \\ E & 0 \end{pmatrix} \ . \tag{3.2}$$

The theories that are of interest to us[3] have a quasi-homogeneous superpotential $W(x_i)$,

$$W\left(\lambda^{w_1} x_1, \ldots, \lambda^{w_n} x_n\right) = \lambda^H \, W\left(x_1, \ldots, x_n\right) \quad \text{for} \quad \lambda \in \mathbb{C} \ , \tag{3.3}$$

ensuring that the bulk theory is superconformal rather than just supersymmetric. More specifically, the A-type Gepner models correspond to polynomials of Fermat type; this means that all w_i divide H so that W takes the form (we restrict our discussion to the case of five variables)

$$W = x_1^{h_1} + x_2^{h_2} + x_3^{h_3} + x_4^{h_4} + x_5^{h_5} \ , \tag{3.4}$$

where $h_i = H/w_i$.

The factorisations Q that are in one-to-one correspondence to the *superconformal* D-branes must also respect this U(1) symmetry; this implies that the entries of Q must be polynomials in the x_i, and furthermore, that there exist matrices $R(\lambda)$ so that

$$R(\lambda)\, Q(\lambda^{w_i} x_i)\, R(\lambda)^{-1} = \lambda^{\frac{H}{2}}\, Q(x_i) \ . \tag{3.5}$$

The matrices $R(\lambda)$ as well as $R(\lambda)^{-1}$ may depend on the x_i in a polynomial way.

[3]The matrix factorisation description of supersymmetric D-branes also applies to more general classes of Landau-Ginzburg models — see for example [28] for a recent discussion in the context of the N=2 sine-Gordon model.

To make contact with the Gepner models we have to consider an orbifold of the Landau-Ginzburg model, where the orbifold group $\mathbb{Z}_H$ acts on the variables x_i as

$$x_i \mapsto \omega^{w_i} x_i \ , \qquad \text{where} \qquad \omega = e^{\frac{2\pi i}{H}} \ . \tag{3.6}$$

This orbifold action must also be implemented on the open strings, and thus we need to choose an action of the orbifold group on Q. More precisely, we need to choose a matrix γ (that together with its inverse is polynomial[4] in the x_i) such that Q satisfies the equivariance condition

$$\gamma \, Q \, (\omega^{w_i} x_i) \, \gamma^{-1} = Q(x_i) \ , \tag{3.7}$$

where $\gamma^H = \mathbf{1}$. If such a γ exists, it is not unique, since we can always multiply a given γ by an H^{th} root of unity. A D-brane in the orbifold theory is thus characterised by Q, together with a choice of representation γ.

Given two D-branes described by (Q, γ) and $(\widehat{Q}, \hat{\gamma})$, the open string spectrum between them is determined by a suitably defined equivariant cohomology. More precisely, the bosons are the maps of the form

$$\phi = \begin{pmatrix} \phi_0 & 0 \\ 0 & \phi_1 \end{pmatrix} \tag{3.8}$$

that are invariant under the orbifold action, i.e.

$$\phi(x_i) = \hat{\gamma} \; \phi \, (\omega^{w_i} x_i) \;\; \gamma^{-1} \ , \tag{3.9}$$

and satisfy the BRST-closure condition

$$\widehat{Q} \, \phi = \phi \, Q \ . \tag{3.10}$$

These bosons are only considered modulo the BRST-exact solutions

$$\tilde{\phi} = \widehat{Q} \, t + t \, Q \ , \qquad \text{where} \qquad t = \begin{pmatrix} 0 & t_1 \\ t_0 & 0 \end{pmatrix} \tag{3.11}$$

describes a fermion. Similarly, the fermions are the invariant maps t that satisfy the BRST-closure condition $\widehat{Q} \, t + t \, Q = 0$, modulo the BRST-exact solutions, $\tilde{t} = \widehat{Q}\phi - \phi \, Q$. Finally, the index between two such D-branes is the number of bosonic states minus the number of fermionic states. For example, for a single minimal model, the self-intersection matrix for the matrix factorisation for which J is linear in x is simply [17]

$$I = (\mathbf{1} - G^{-w}) \ , \tag{3.12}$$

where G is the H-dimensional shift matrix, and $w = H/h$. This H-dimensional intersection matrix accounts for the H different choices for the matrix γ in (3.7) that are obtained from a given γ by multiplication by an H^{th} root of unity.

Finally, as was explained in [17, 19], one can tensor matrix factorisations together: if Q_1 and Q_2 are matrix factorisations for the superpotentials W_1 and W_2, respectively, the

[4]Note that in all examples which we shall discuss γ is constant.

tensor product factorisation $Q_1 \widehat{\otimes} Q_2$ is a matrix factorisation for W_1+W_2. Furthermore, the intersection matrix for the tensor product factorisation is just the product of the separate intersection matrices. For example, for the five-fold tensor product factorisation of the superpotential (3.4), the self-intersection matrix of the factorisations for which each J_i is linear is simply

$$I_{\rm RS} = \prod_{i=1}^{5} (\mathbf{1} - G^{-w_i}) \ . \tag{3.13}$$

This then agrees with the Witten index of the open string spectrum of the tensor product (or Recknagel-Schomerus) branes with $L_i = 0$ [17].

3.2 Generalised permutation factorisations

It was shown in [14] that the transposition branes (i.e. the branes where the permutation is just a transposition between the two factors $k_1 = k_2$ say) correspond to tensor products of factorisations that involve the rank 1 factorisation in the first two factors ($d = k_1 + 2 = k_2 + 2$)

$$W_{12} = x_1^d + x_2^d = J\, E \ , \qquad \text{where} \qquad J = \prod_{\eta\in\mathcal{I}} (x_1 - \eta x_2) \ , \qquad E = \prod_{\eta'\notin\mathcal{I}} (x_1 - \eta' x_2) \ , \tag{3.14}$$

as well as the usual factorisations for x_3, x_4 and x_5. Here the η's run over the d different $d^{\rm th}$ roots of -1, and $\mathcal{I}$ denotes some (suitable) subset of these roots. Such factorisations were first considered in [17]. The generalisation for higher order permutations was found in [16].

It is relatively obvious how this construction can be generalised to the case where the two exponents are not the same, but only contain a non-trivial common factor. To set up notation, let us consider a superpotential in two factors $W = x_1^{h_1} + x_2^{h_2} = x_1^{dr_1} + x_2^{dr_2}$, where $d \geq 2$ is the greatest common divisor of h_1 and h_2, $d = \gcd(h_1, h_2)$. We can obviously factorise

$$W = x_1^{dr_1} + x_2^{dr_2} = \prod_\eta (x_1^{r_1} - \eta x_2^{r_2}) \ , \tag{3.15}$$

where η is in turn each of the d different $d^{\rm th}$ roots of -1. Then we can define a rank 1 factorisation by taking J to be the product of some of these factors, with E being the product of the remaining factors. We shall call these rank 1 factorisations *generalised permutation factorisations*, and we shall sometimes denote them by $(\widetilde{12})$. By the same arguments as in [14] one can show that the factorisations where J or E contains more than one factor can be obtained as bound states of those where either J or E is a single factor. For the analysis of the charges it should therefore be sufficient to consider these factorisations only, and this is indeed what we shall find. In the following we thus concentrate on factorisations for which J consists of a single factor, $J = (x_1^{r_1} - \eta x_2^{r_2})$. They will be denoted by Q_η.

For the analysis of the charges it is important to determine the corresponding open string spectra, and in particular the index. This will be done next.

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3.3 Spectra and indices

The calculations to determine the spectra and indices are all relatively straightforward, so we shall only explain them in one example and give the results for the remaining cases. The simplest case is the open string spectrum between two branes corresponding to generalised permutation factorisations.

3.3.1 Spectrum of generalised permutation factorisations

The discussion of the spectrum between two factorisations Q_η and $\widehat{Q}_{\hat\eta}$ depends on whether η and $\hat\eta$ coincide or not, so we shall distinguish these two cases.

The case $\eta \neq \hat\eta$

The BRST-closure condition for the bosons $\widehat{Q}_{\hat\eta}\,\phi - \phi\,Q_\eta = 0$ gives

$$\begin{aligned} \phi_1\,(x_1^{r_1} - \hat\eta x_2^{r_2}) - \phi_0\,(x_1^{r_1} - \eta x_2^{r_2}) = 0 \\ \phi_0 \prod_{\hat\eta' \neq \hat\eta} (x_1^{r_1} - \hat\eta' x_2^{r_2}) - \phi_1 \prod_{\eta' \neq \eta} (x_1^{r_1} - \eta' x_2^{r_2}) = 0\,. \end{aligned} \tag{3.16}$$

Since $\eta \neq \hat\eta$, the unique solution is

$$\phi_0 = a\,(x_1^{r_1} - \hat\eta x_2^{r_2}) \qquad \text{and} \qquad \phi_1 = a\,(x_1^{r_1} - \eta x_2^{r_2})\,, \tag{3.17}$$

where $a \in \mathbb{C}[x_1, x_2]$ is an arbitrary polynomial. The bosonic operator is BRST-exact if $\phi = \widehat{Q}_{\hat\eta}\,t + t\,Q_\eta$ for an arbitrary fermionic operator t. It is easy to see that every solution (3.17) is BRST-trivial and thus there are no bosons propagating between two such branes.

For the fermions, the BRST-closure condition gives

$$\begin{aligned} t_0\,(x_1^{r_1} - \hat\eta x_2^{r_2}) + t_1 \prod_{\eta' \neq \eta} \left(x_1^{r_1} - \eta' x_2^{r_2}\right) = 0 \\ t_1 \prod_{\hat\eta' \neq \hat\eta} \left(x_1^{r_1} - \hat\eta' x_2^{r_2}\right) + t_0\,(x_1^{r_1} - \eta x_2^{r_2}) = 0\,. \end{aligned} \tag{3.18}$$

This is solved by

$$t_1 \in \mathbb{C}[x_1, x_2] \quad \text{and} \quad t_0 = -t_1 \prod_{\eta' \neq \eta, \hat\eta} \left(x_1^{r_1} - \eta' x_2^{r_2}\right)\,, \tag{3.19}$$

so the BRST-closed operators are determined by t_1. The fermionic operator is BRST-exact if $t = \widehat{Q}_{\hat\eta}\,\phi - \phi\,Q_\eta$ for an arbitrary bosonic operator ϕ. This is the case if t_1 lies in the ring

$$t_1 \in \langle (x_1^{r_1} - \hat\eta x_2^{r_2})\,, (x_1^{r_1} - \eta x_2^{r_2}) \rangle\,. \tag{3.20}$$

Representatives of the BRST cohomology can thus be chosen as

$$t^{(i,j)}\,(x_1, x_2) = \begin{pmatrix} 0 & x_1^i x_2^j \\ -x_1^i x_2^j \prod\limits_{\eta' \neq \eta, \hat\eta} (x_1^{r_1} - \eta' x_2^{r_2}) & 0 \end{pmatrix}\,, \tag{3.21}$$

where

$$i = 0, \ldots, r_1 - 1 \quad \text{and} \quad j = 0, \ldots, r_2 - 1 \ . \tag{3.22}$$

The number of fermions propagating between the branes is therefore $r_1 r_2$. The equivariance condition (3.7) for Q_η is satisfied by choosing γ to be one of the matrices

$$\gamma_\mu = \begin{pmatrix} \omega^{-\frac{H/d+\mu}{2}} & 0 \\ 0 & \omega^{\frac{H/d-\mu}{2}} \end{pmatrix} \ , \tag{3.23}$$

where μ is an integer defined mod $2H$ that has the property that $H/d+\mu$ is even (similarly one defines $\hat{\gamma}_{\hat{\mu}}$ with an integer $\hat{\mu}$). One then easily calculates that the above fermionic operators transform as

$$\hat{\gamma}_{\hat{\mu}} \ t^{(i,j)}(\omega^{w_l} x_l) \ \gamma_\mu^{-1} = \omega^{-\frac{H}{d}+w_1 i+w_2 j+\frac{\mu-\hat{\mu}}{2}} \ t^{(i,j)}(x_l) \ , \tag{3.24}$$

where $i = 0, \ldots, r_1 - 1$ and $j = 0, \ldots, r_2 - 1$. We can thus express the index in terms of the H-dimensional shift matrix G

$$I_{(\widetilde{12})} = -G^{-\frac{H}{d}} \left(\mathbf{1} + G^{w_1} + \cdots + G^{(r_1-1)w_1} \right) \left(\mathbf{1} + G^{w_2} + \cdots + G^{(r_2-1)w_2} \right) \ . \tag{3.25}$$

The case $\eta = \hat{\eta}$

For the case $\eta = \hat{\eta}$ one easily shows that there are no fermions. For the bosons, one can take the representatives of the BRST cohomology to be given by

$$\phi^{(i,j)}(x_1, x_2) = \begin{pmatrix} x_1^i x_2^j & 0 \\ 0 & x_1^i x_2^j \end{pmatrix} \ , \tag{3.26}$$

with

$$i = 0, \ldots, r_1(d-1) - 1 \qquad \text{and} \qquad j = 0, \ldots, r_2 - 1 \ . \tag{3.27}$$

Thus the number of bosons propagating in the self-overlap is equal to $r_1 r_2 \, (d-1)$. With respect to the above choice of γ_μ the bosonic fields transform as

$$\hat{\gamma}_{\hat{\mu}} \, \phi^{(i,j)} \, (\omega^{w_l} x_l) \ \gamma_\mu^{-1} = \omega^{w_1 i + w_2 j + \frac{\mu-\hat{\mu}}{2}} \, \phi^{(i,j)} \, (x_l) \ , \tag{3.28}$$

and therefore the intersection matrix reads

$$I_{(\widetilde{12})} = \left(\mathbf{1} + G^{w_1} + \cdots + G^{(r_1(d-1)-1)w_1} \right) \left(\mathbf{1} + G^{w_2} + \cdots + G^{(r_2-1)w_2} \right) \ . \tag{3.29}$$

Before we move on let us take a look at how our formulae simplify when we consider the actual permutation case, $w_1 = w_2$. Then we have $d = h_1 = h_2$ and thus $r_1 = r_2 = 1$. If $\eta \neq \hat{\eta}$, the intersection matrix (3.25) takes the simple form

$$I_{(12)} = -G^{-\frac{H}{d}} = -G^{-w_1} \qquad (\eta \neq \hat{\eta}) \ . \tag{3.30}$$

For $\eta = \hat{\eta}$, on the other hand, the intersection matrix (3.29) reads

$$I_{(12)} = \left(\mathbf{1} + G^{w_1} + \cdots + G^{(d-2)w_1} \right) \qquad (\eta = \hat{\eta}) \ . \tag{3.31}$$

The formulae for these special cases were already given in [17].

3.3.2 Relative intersection forms

In order to determine the charges it is also important to calculate the intersection form between these generalised permutation factorisations and the tensor product factorisations. Using similar techniques one finds that there is always one boson and one fermion between these two branes. Furthermore, the U(1) charges are such that the intersection matrix is

$$I_{\mathrm{RS}-(\widetilde{12})} = G^{-\lfloor \frac{w_1}{2} \rfloor - \lfloor \frac{w_2}{2} \rfloor} \left(G^{\lfloor -\frac{H}{2d} \rfloor} - G^{\lfloor \frac{H}{2d} \rfloor} \right) \ , \tag{3.32}$$

where $\lfloor n \rfloor$ denotes the Gauss bracket, i.e. the greatest integer less or equal to n.

For the case of the actual permutation branes these formulae reproduce again those of [17]. We also note that the morphisms and the intersection matrices are in fact independent of the choice of η.

Finally, we need two further classes of relative intersection forms for our analysis of the Gepner models. If the superpotential is of the form

$$W = x_1^{h_1} + x_2^{h_2} + x_3^{h_3} \ , \qquad d_i = \mathrm{gcd}(h_i, h_{i+1}) \geq 2 \ , \tag{3.33}$$

then the intersection matrix between the two generalised permutation factorisations corresponding to $(\widetilde{12})$ and $(\widetilde{23})$ is [29]

$$\begin{aligned} I_{(\widetilde{12})-(\widetilde{23})} = {} & G^{-\lfloor \frac{H}{2d_1} \rfloor + \lfloor -\frac{H}{2d_2} \rfloor + \lfloor -\frac{w_1}{2} \rfloor - \lfloor \frac{w_3}{2} \rfloor} \\ & \times \left(\mathbf{1} + G^{w_2} + \cdots + G^{w_2(\min(r,s)-1)} \right) \left(\mathbf{1} - G^{w_2 \max(r,s)} \right) \ , \end{aligned} \tag{3.34}$$

where $r = h_2/d_1$ and $s = h_2/d_2$.

If the superpotential is of the form

$$W = x_1^{h_1} + x_2^{h_2} + x_3^{h_3} + x_4^{h_4} \ , \qquad d_i = \mathrm{gcd}(h_i, h_{i+1}) \geq 2 \ , \tag{3.35}$$

and we define $r_1 = h_2/d_1$, $r_2 = h_3/d_2$, then the intersection matrix between the two generalised permutation factorisations $(\widetilde{12})(\widetilde{34})$ and $(\widetilde{23})$ is [29]

$$\begin{aligned} I_{(\widetilde{12})(\widetilde{34})-(\widetilde{23})} = {} & G^{-\lfloor \frac{H}{2d_1} \rfloor - \lfloor \frac{H}{2d_3} \rfloor + \lfloor -\frac{H}{2d_2} \rfloor + \lfloor -\frac{w_1}{2} \rfloor + \lfloor -\frac{w_4}{2} \rfloor} \left(\mathbf{1} + G^{w_2} + \cdots + G^{w_2(r_1-1)} \right) \\ & \times \left(\mathbf{1} + G^{w_3} + \cdots + G^{w_3(r_2-1)} \right) \left(\mathbf{1} - G^{\frac{H}{d_3}} \right) \ , \end{aligned} \tag{3.36}$$

where we spell out for simplicity only the case where $d_1 \geq d_2 \geq d_3$, since the other cases will not be relevant for our analysis of the Gepner models.

3.4 RR charges from path integrals

There is an alternative way to determine the intersection matrix by computing directly the RR-charges of the factorisations, i.e. the topological correlators of one bulk-field in the presence of a boundary. For Landau-Ginzburg models with the boundary condition described by a matrix factorisation, these correlators can be determined using path integral methods [9]. A generalisation to orbifolds has been proposed in [30]. Before we spell out the concrete expressions of [30], we need to introduce some notation. We label the RR ground states $|n;\alpha\rangle$ in the n^{th} twisted sector by a multi-index $\alpha = (\alpha_1, \ldots, \alpha_{u_n})$. Here, u_n is the number of factors for which $n = 0 \mod h_i$ and we assume that we have re-ordered the factors such that these are the first u_n ones. The state $|n;\alpha\rangle$ is the ground state in the sector

$$\bigotimes_{i=1}^{u_n} \mathcal{H}_{(\alpha_i,-\alpha_i-1,-1)} \otimes \bar{\mathcal{H}}_{(\alpha_i,-\alpha_i-1,-1)} \otimes \bigotimes_{j=u_n+1}^{5} \mathcal{H}_{(n_j-1,n_j,1)} \otimes \bar{\mathcal{H}}_{(n_j-1,-n_j,-1)}\Big|_{n_j=n \mod h_i} . \tag{3.37}$$

It is obtained from the state $|n;0\rangle$ by acting with the field $\phi_n^\alpha = \prod_{i=1}^{u_n} x_i^{\alpha_i}$.

The charge of the factorisation Q (together with a representation γ of the orbifold group) under the field corresponding to $|n;\alpha\rangle$ is then given by [30]

$$\mathrm{ch}(Q,\gamma)(|n;\alpha\rangle) = \frac{1}{u_n!}\mathrm{Res}_{W_n}(\phi_n^\alpha\, \mathrm{Str}[\gamma^n (\partial Q)^{\wedge u_n}]) . \tag{3.38}$$

Here Str denotes the supertrace, i.e. the difference between the trace of the upper left and the lower right $r \times r$ block of the $2r \times 2r$ matrix in the bracket, and the residue is defined as

$$\mathrm{Res}_{W_n}(f(x_i)) = \frac{1}{(2\pi i)^{u_n}} \oint \frac{f(x_i)}{\partial_1 W \cdots \partial_{u_n} W}\Big|_{x_j=0\ (j>u_n)} dx_1 \cdots dx_{u_n} . \tag{3.39}$$

The expressions for the charges allow one to determine the intersection index as [30]

$$\mathrm{Tr}(-1)^F = \Big\langle \mathrm{ch}(\widehat{Q},\hat{\gamma}), \mathrm{ch}(Q,\gamma) \Big\rangle , \tag{3.40}$$

where

$$\Big\langle \mathrm{ch}(\widehat{Q},\hat{\gamma}), \mathrm{ch}(Q,\gamma) \Big\rangle = \frac{1}{H}\sum_{n=0}^{H-1}\sum_{\alpha,\beta} \mathrm{ch}(\widehat{Q},\hat{\gamma})(|n;\alpha\rangle) \frac{1}{\prod_{\frac{n}{h_i}\notin\mathbb{Z}}(1-\omega^{w_i n})} \eta_n^{\alpha\beta^*} \mathrm{ch}(Q,\gamma)(|n;\beta\rangle)^* . \tag{3.41}$$

Here, $\eta_n^{\alpha\beta}$ is the inverse of the closed topological string metric,

$$\eta^n_{\alpha\beta} = \mathrm{Res}_{W_n}(\phi_n^\alpha \phi_n^\beta) \tag{3.42}$$

and β^* denotes the label of the field conjugate to ϕ_n^β (which is given by $\beta_i^* = h_i - 2 - \beta_i$). Note that the formula (3.41) has been slightly modified in comparison to [30] by introducing the conjugate label.

For the factorisations we are considering here, namely products of generalised permutation factorisations and tensor product factorisations, the charge formula (3.38) factorises, so we can focus on the case of just two factors and a generalised permutation factorisation Q_η. In the untwisted sector ($n = 0 \mod r_1 r_2 d$), the charge is given by

$$\mathrm{ch}(Q_\eta,\gamma)(|n;(r_1 m - 1, r_2(d-m)-1)\rangle) = \frac{1}{d}\eta^m , \tag{3.43}$$

where $m = 1, \ldots, d-1$. In the twisted sectors we only have a non-zero contribution if $n \neq 0 \mod r_i d$ for $i = 1, 2$, and it is given by

$$\mathrm{ch}(Q_\eta,\gamma)(|n;0\rangle) = \mathrm{Str}(\gamma^n) . \tag{3.44}$$

Using these results in (3.40), we have checked in all relevant examples that the result agrees with the intersection matrices obtained in section 3.3.

4. Application to Gepner models

After these preparations we are now in a position to analyse the 147 different Fermat type Calabi-Yau manifolds in detail. Before we begin with this analysis we should explain more precisely what we expect, and what we are looking for.

4.1 RR vector spaces and RR lattices

The D-branes we are interested in carry RR charges. In general, different D-branes carry different RR charges, and there is a whole vector space of such charges. The dimension of this vector space equals the dimension of the even cohomology of the corresponding Calabi-Yau manifold. (Since we are only considering B-type D-branes here, only the even dimensional cohomology is of relevance.) For all the 147 Fermat type Calabi-Yau manifolds this dimension is known [31, 32, 24].[5]

One natural objective we may have is to find a set of D-branes whose charges form a basis for this vector space. This is what we mean by 'spanning the RR vector space' in the following. From the conformal field theory point of view, this question was analysed in section 2, where we saw that for 31 models the RR vector space is not spanned by tensor product or permutation branes. From the point of view of the matrix factorisations, the condition that a set of factorisations spans the RR vector space can also be easily formulated: it simply means that the rank of its intersection matrix agrees with the dimension of the even cohomology. With the explicit formulae for the intersection matrices, this condition can be easily tested in each case. Given that we know which factorisations correspond to tensor product and permutation branes, we thus expect that there are 31 models for which these factorisations do not span the RR vector space.

The RR charges of these theories do not just form a vector space, but they actually form a lattice. This reflects the fact that RR charges are quantised. We can therefore ask a more detailed question: which D-branes do not just generate the RR vector space, but actually 'span the RR lattice'. From the conformal field theory point of view, this question is not straightforward, since we do not know how to describe the D-branes that span the RR vector space for 31 models. However, from the point of view of the matrix factorisations this question can again be easily analysed: a set of factorisations spans the RR lattice if its intersection matrix contains a submatrix with maximal rank (equalling the dimension of the even cohomology) that has determinant equal to 1! Given the explicit formulae for the various intersection matrices we can fairly systematically look for factorisations which have this property.

4.2 Explicit results

With this in mind we have analysed the matrix factorisations for all 147 models. In each case we have first tried to find tensor product and conventional permutation factorisations that span the RR vector space. This was possible in all but 32 cases; these 32 cases are described in table 2.

[5] See also the URL `http://hep.itp.tuwien.ac.at/~kreuzer/CY`.

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Calabi-Yau	Gepner model	RR dim	RR vector space	RR charge lattice
$\mathbb{P}_{(3,3,4,6,8)}[24]$	$(6,6,4,2,1)$	16	$(\widetilde{35})_{0,2}$	$(12)(\widetilde{35})_{0,2}$
$\mathbb{P}_{(1,3,6,10,10)}[30]$	$(28,8,3,1,1)$	40	$(\widetilde{23})(45)_{0,2,4,6;0,2}$	$(\widetilde{23})(45)_{0,2,4,6;0,2}$
$\mathbb{P}_{(3,5,6,6,10)}[30]$	$(8,4,3,3,1)$	32	$(34)(\widetilde{25})_{0,2,4,6;0,2}$	$(34)(\widetilde{25})_{0,2,4,6;0,2}$
$\mathbb{P}_{(1,1,4,12,18)}[36]$	$(34,34,7,1,0)$	16	$(\widetilde{34})_{0,2}$	$(12)(\widetilde{34})_{0,2}$
$\mathbb{P}_{(2,4,9,9,12)}[36]$	$(16,7,2,2,1)$	40	$(\widetilde{25})(34)_{0,2,4}$	$(\widetilde{25})(34)_{0,2,4}$
$\mathbb{P}_{(1,3,12,16,16)}[48]$	$(46,14,2,1,1)$	54	$(\widetilde{23})(45)_{0,2,4}$	$(\widetilde{23})(45)_{0,2,4}$
$\mathbb{P}_{(1,2,6,18,27)}[54]$	$(52,25,7,1,0)$	22	$(\widetilde{34})_{0,2}$	$(\widetilde{34})_{0,2}$
$\mathbb{P}_{(1,4,5,20,30)}[60]$	$(58,13,10,1,0)$	24	$(\widetilde{24})_{0,2}$	$(\widetilde{13})(\widetilde{24})_{0,2}$
$\mathbb{P}_{(2,3,15,20,20)}[60]$	$(28,18,2,1,1)$	64	$(\widetilde{23})(45)_{0,2,4}$	$(\widetilde{23})(45)_{0,2,4}$
$\mathbb{P}_{(3,3,4,20,30)}[60]$	$(18,18,13,1,0)$	24	$(\widetilde{34})_{0,2}$	$(12)(\widetilde{34})_{0,2}$
$\mathbb{P}_{(3,5,12,20,20)}[60]$	$(18,10,3,1,1)$	64	$(\widetilde{13})(45)_{0,2,4,6;0,2}$	$(\widetilde{13})(45)_{0,2,4,6;0,2}$
$\mathbb{P}_{(3,10,12,15,20)}[60]$	$(18,4,3,2,1)$	32	$(\widetilde{25})_{0,2}$	$(\widetilde{25})_{0,2}$
$\mathbb{P}_{(4,6,15,15,20)}[60]$	$(13,8,2,2,1)$	56	$(\widetilde{15})(34)_{0,2,4}$	$(\widetilde{15})(34)_{0,2,4}$
$\mathbb{P}_{(1,3,8,24,36)}[72]$	$(70,22,7,1,0)$	32	$(\widetilde{34})_{0,2}$	$(\widetilde{12})(\widetilde{34})_{0,2}$
$\mathbb{P}_{(1,6,21,28,28)}[84]$	$(82,12,2,1,1)$	82	$(45)_{0,2}-(\widetilde{23})$	$(\widetilde{12})(45)_{0,2}-(\widetilde{23})(45)$
$\mathbb{P}_{(3,4,7,28,42)}[84]$	$(26,19,10,1,0)$	34	$(15)(\widetilde{24})_{0,2}$	$(15)(\widetilde{24})_{0,2}$
$\mathbb{P}_{(3,4,21,28,28)}[84]$	$(26,19,2,1,1)$	84	$(\widetilde{13})(45)_{0,2,4}$	$(\widetilde{13})(45)_{0,2,4}$
$\mathbb{P}_{(2,3,10,30,45)}[90]$	$(43,28,7,1,0)$	38	$(\widetilde{34})_{0,2}$	$(\widetilde{34})_{0,2}$
$\mathbb{P}_{(1,4,20,25,50)}[100]$	$(98,23,3,2,0)$	68	$(\widetilde{23})(45)_{0,2,4,6;0}$	$(\widetilde{23})(45)_{0,2,4,6;0}$
$\mathbb{P}_{(1,6,14,42,63)}[126]$	$(124,19,7,1,0)$	58	$(\widetilde{34})_{0,2}-(\widetilde{24})_{0,2}$	$(\widetilde{34})_{0,2}-(\widetilde{24})_{0,2}$
$\mathbb{P}_{(1,9,20,60,90)}[180]$	$(178,18,7,1,0)$	86	$(25)(\widetilde{34})_{0,2}$	$(25)(\widetilde{34})_{0,2}$
$\mathbb{P}_{(4,5,36,45,90)}[180]$	$(43,34,3,2,0)$	112	$(\widetilde{13})(45)_{0,2,4,6;0}$	$(\widetilde{13})(45)_{0,2,4,6;0}$
$\mathbb{P}_{(2,9,22,66,99)}[198]$	$(97,20,7,1,0)$	92	$(25)(\widetilde{34})_{0,2}$	$(25)(\widetilde{34})_{0,2}$
$\mathbb{P}_{(6,14,15,70,105)}[210]$	$(33,13,12,1,0)$	88	$(\widetilde{24})(35)_{0,2}$	$(\widetilde{24})(35)_{0,2}$
$\mathbb{P}_{(1,8,27,72,108)}[216]$	$(214,25,6,1,0)$	98	$(\widetilde{24})(35)_{0,2}$	$(\widetilde{24})(35)_{0,2}$
$\mathbb{P}_{(3,8,33,88,132)}[264]$	$(86,31,6,1,0)$	116	$(\widetilde{24})(35)_{0,2}$	$(\widetilde{24})(35)_{0,2}$
$\mathbb{P}_{(1,6,42,98,147)}[294]$	$(292,47,5,1,0)$	96	$(\widetilde{23})_{0,\ldots,10}$	$(\widetilde{23})_{0,\ldots,10}$
$\mathbb{P}_{(3,22,30,110,165)}[330]$	$(108,13,9,1,0)$	120	$(\widetilde{24})_{0,2}$	$(\widetilde{24})_{0,2}$
$\mathbb{P}_{(1,18,38,114,171)}[342]$	$(340,17,7,1,0)$	144	$(\widetilde{34})_{0,2}$	$(\widetilde{34})_{0,2}$
$\mathbb{P}_{(6,7,78,182,273)}[546]$	$(89,76,5,1,0)$	168	$(\widetilde{13})_{0,\ldots,10}$	$(\widetilde{13})_{0,\ldots,10}$
$\mathbb{P}_{(1,14,90,210,315)}[630]$	$(628,43,5,1,0)$	192	$(\widetilde{24})_{0,2}$	$(\widetilde{24})_{0,2}$
$\mathbb{P}_{(3,14,102,238,357)}[714]$	$(236,49,5,1,0)$	216	$(\widetilde{24})_{0,2}$	$(\widetilde{24})_{0,2}$

Table 2: Models that require generalised permutation factorisations in order to span the RR vector space.

Most of the conventions used in this table should be clear; a more detailed explanation of the conventions is given in the appendix.

This list of models agrees with that of table 1, except for the model $\mathbb{P}_{(1,6,21,28,28)}[84]$, for which the matrix factorisation analysis predicts that a generalised permutation factorisation is required in the factor $(\widetilde{23})$, while the conformal field theory analysis predicts that all RR charges can be accounted for in terms of the usual tensor product and permutation branes. This mismatch is easily resolved: the generalised permutation factorisation $(\widetilde{23})$ is of the form

$$Q\ :\qquad J = x_2^{h_2/2} + i x_3^{h_3/2}\ ,\qquad E = x_2^{h_2/2} - i x_3^{h_3/2}\ ,\tag{4.1}$$

or

$$Q^r\ :\qquad J = x_2^{h_2/2} - i x_3^{h_3/2}\ ,\qquad E = x_2^{h_2/2} + i x_3^{h_3/2}\ .\tag{4.2}$$

These two factorisations describe anti-branes of one another, and one can easily see that their direct sum $Q \oplus Q^r$ is equivalent (in the sense of [33]) to the tensor product factorisation with $J_2 = E_2 = x_2^{h_2/2}$, $J_3 = E_3 = x_3^{h_3/2}$. Thus these two factorisations describe in fact the two resolved (short orbit) tensor product D-branes!

This identification of certain rank 1 factorisations with resolved tensor product D-branes actually occurs more generally, whenever d (and thus h_2 and h_3) is even. Then one always has the two rank 1 factorisations (4.1) and (4.2) as above, and by the same argument they always correspond to resolved tensor product D-branes. In general, these factorisations are just two special cases of a whole set of (generalised) permutation factorisations; in fact they correspond to the factorisations where J contains precisely every second $d^{\rm th}$ root of -1 (while E is the product of the other factors). What is special about the case when $d = 2$ is the only common divisor, is that *all* possible generalised permutation factorisations are of this type.

The simplest case for which all the 'generalised permutation factorisations' correspond to resolved tensor product branes therefore occurs when one of the two factors has $h = 2$ (and thus $k = 0$). This is in fact also clear from the conformal field theory perspective since the level $k = 0$ factor is trivial, and thus cannot be involved in any 'new' construction. Since these factorisations appear quite frequently and are obviously not new, we have treated them as conventional tensor product constructions. Apart from these cases, the model $\mathbb{P}_{(1,6,21,28,28)}[84]$ is the only example where a generalised permutation factorisation with $d = 2$ occurs. In particular, it is then clear that for the remaining 31 theories the generalised permutation factorisations cannot be interpreted as resolved tensor product branes. Thus the matrix factorisation analysis agrees beautifully with the conformal field theory analysis of section 2. However, now we can actually specify the D-branes that are required to account for the charges: they are described by generalised permutation factorisations. Given the similarity to the usual permutation factorisations, it seems appropriate to call the corresponding branes 'generalised permutation branes'.

From the point of view of the matrix factorisation description we can also analyse which factorisations are required in order to span the RR charge lattice (not just the RR vector space). We have analysed this question for all the 147 models in detail, and we have found that the only factorisations that are required in addition to the tensor product and permutation factorisations are again the generalised permutation factorisations described above. The actual constructions that do the job in all cases are spelled out in table 2 and table 3 (in the appendix).

5. Summary

In summary, we have therefore shown that in general the tensor product and permutation branes do not account for all RR charges of Gepner models. This result could be obtained using either a direct conformal field theory analysis, or the results from the matrix factorisations description of D-branes. Using the latter approach, we could furthermore identify, at least for the 147 Fermat type Calabi-Yau manifolds, what additional constructions are needed: the only additional D-branes that are required are generalised permutation branes that should exist whenever the relevant (shifted) levels have a non-trivial common factor. We were also able to show that the corresponding generalised permutation factorisations generate the full RR charge lattice.

The generalised permutation branes that appear in our analysis are very reminiscent of the generalised permutation branes for products of SU(2) WZW-models for which evidence was recently found in [20]. The matrix factorisation description also determines various properties of these D-branes, such as their charges, the topological open string spectrum, *etc.* This should help to construct these D-branes in conformal field theory.

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A. The generating matrix factorisations

In this appendix we describe the matrix factorisations that span the RR vector space and the RR charge lattice for all $147-32$ A-type Gepner models. (The remaining 32 cases were already described in table 2.)

In the following table, the first three entries should be self-explanatory. In the fourth and fifth column we have described the matrix factorisations that generate the RR vector space, and the RR charge lattice, respectively. A few words containing conventions are in order.

(i) RS stands for Recknagel-Schomerus construction, i.e. for the simplest tensor product factorisations whose intersection form was given at the end of section 3.1.

(ii) Similarly, (12) stands for the (12)-permutation factorisation (with tensor product factorisations for the factors $3, 4$ and 5). In the same vain, (12)(34) denotes the permutation factorisation, where we have a transposition in the factors (12) and the factors (34), *etc.*

(iii) By $(45)_{0,2}$ we mean that one has to consider the (45)-permutation factorisations with two different values for η: $\eta = e^{\frac{-\pi i(M+1)}{d}}$ with $M = 0, 2$.
Similarly, by $(23)(45)_{0,\ldots,10;0,2}$ we mean that one takes the (23)-permutation factorisation with $M_{23} = 0, 2, \ldots, 10$, and the (45)-permutation factorisation with $M_{45} = 0, 2$. (For ease of notation we write $(23)(45)_{0,2,4}$ for $(23)(45)_{0,2,4;0,2,4}$.)

(iv) By (45)−(35) we mean the set of factorisations which have a (45)-permutation factorisation (but are of tensor type in the first, second and third factor), together with the set of factorisations that are (35)-permutation but tensor in the first, second and fourth factor.

We have furthermore denoted generalised permutation factorisations by a tilde; for example $(\widetilde{12})$ denotes the generalised permutation factorisation in the first two factors. There is one exception to this rule: as was explained at the end of section 4.2, the 'generalised' permutation factorisations that involve a trivial factor with $k = 0$ (usually the fifth factor), do not describe novel D-branes, but rather correspond to resolved tensor product branes. Thus for example in the model $\mathbb{P}_{(1,1,1,3,6)}[12]$, whose corresponding levels are $(k_1, k_2, k_3, k_4, k_5) = (10, 10, 10, 2, 0)$ the generalised permutation factorisation involving the last two factors describes in fact a usual tensor product brane.

The models included below have the property that the RR vector space is spanned by tensor product or permutation branes. However, even for these models it is clear that some require generalised permutation branes to account for the full RR charge lattice!

Calabi-Yau	Gepner model	RR dim	RR vector space	RR charge lattice
$\mathbb{P}_{(1,1,1,1,1)}[5]$	$(3,3,3,3,3)$	4	RS	(12)(34)
$\mathbb{P}_{(1,1,1,1,2)}[6]$	$(4,4,4,4,1)$	4	RS	(12)(34)
$\mathbb{P}_{(1,1,1,1,4)}[8]$	$(6,6,6,6,0)$	4	RS	(12)(34)
$\mathbb{P}_{(1,1,2,2,2)}[8]$	$(6,6,2,2,2)$	6	RS	(12)(34)
$\mathbb{P}_{(1,1,1,3,3)}[9]$	$(7,7,7,1,1)$	10	$(45)_{0,2}$	$(12)(45)_{0,2}$
$\mathbb{P}_{(1,1,1,2,5)}[10]$	$(8,8,8,3,0)$	4	RS	RS
$\mathbb{P}_{(1,1,1,3,6)}[12]$	$(10,10,10,2,0)$	8	(45)	(12)(45)
$\mathbb{P}_{(1,1,2,2,6)}[12]$	$(10,10,4,4,0)$	6	RS	(12)(34)
$\mathbb{P}_{(1,1,2,4,4)}[12]$	$(10,10,4,1,1)$	12	$(45)_{0,2}$	$(12)(45)_{0,2}$
$\mathbb{P}_{(1,1,3,3,4)}[12]$	$(10,10,2,2,1)$	12	$(34)_{0,2,4}$	$(12)(34)_{0,2,4}$
$\mathbb{P}_{(1,2,2,3,4)}[12]$	$(10,4,4,2,1)$	6	RS	(23)
$\mathbb{P}_{(1,2,3,3,3)}[12]$	$(10,4,2,2,2)$	8	RS	$(\widetilde{12})(34)$
$\mathbb{P}_{(2,2,2,3,3)}[12]$	$(4,4,4,2,2)$	14	$(45)_{0,2,4}$	$(12)(45)_{0,2,4}$
$\mathbb{P}_{(1,2,2,2,7)}[14]$	$(12,5,5,5,0)$	6	RS	(23)
$\mathbb{P}_{(1,1,3,5,5)}[15]$	$(13,13,3,1,1)$	16	$(45)_{0,2}$	$(12)(45)_{0,2}$
$\mathbb{P}_{(1,3,3,3,5)}[15]$	$(13,3,3,3,1)$	8	RS	(23)
$\mathbb{P}_{(1,1,2,4,8)}[16]$	$(14,14,6,2,0)$	10	(45)	(12)(45)
$\mathbb{P}_{(1,1,1,6,9)}[18]$	$(16,16,16,1,0)$	6	RS	RS
$\mathbb{P}_{(1,2,3,3,9)}[18]$	$(16,7,4,4,0)$	8	RS	(34)
$\mathbb{P}_{(1,2,3,6,6)}[18]$	$(16,7,4,1,1)$	16	$(45)_{0,2}$	$(\widetilde{12})(45)_{0,2}$
$\mathbb{P}_{(2,2,2,3,9)}[18]$	$(7,7,7,4,0)$	10	(45)	(12)(45)
$\mathbb{P}_{(1,1,4,4,10)}[20]$	$(18,18,3,3,0)$	16	$(34)_{0,2,4,6}$	$(12)(34)_{0,2,4,6}$
$\mathbb{P}_{(1,2,2,5,10)}[20]$	$(18,8,8,2,0)$	14	(45)	(23)(45)
$\mathbb{P}_{(1,4,5,5,5)}[20]$	$(18,3,2,2,2)$	12	RS	(34)
$\mathbb{P}_{(2,4,4,5,5)}[20]$	$(8,3,3,2,2)$	32	$(23)(45)_{0,2,4,6}$	$(23)(45)_{0,2,4,6}$

Table 3: (*continued*)

Calabi-Yau	Gepner model	RR dim	RR vector space	RR charge lattice
$\mathbb{P}_{(1,3,3,7,7)}[21]$	$(19,5,5,1,1)$	36	$(23)(45)_{0,\ldots,10;0,2}$	$(23)(45)_{0,\ldots,10;0,2}$
$\mathbb{P}_{(1,1,2,8,12)}[24]$	$(22,22,10,1,0)$	8	RS	RS
$\mathbb{P}_{(1,1,4,6,12)}[24]$	$(22,22,4,2,0)$	18	$(45)-(35)$	$(12)(45)-(12)(35)$
$\mathbb{P}_{(1,1,6,8,8)}[24]$	$(22,22,2,1,1)$	24	$(45)_{0,2}$	$(12)(45)_{0,2}$
$\mathbb{P}_{(1,2,3,6,12)}[24]$	$(22,10,6,2,0)$	14	(45)	$(\widetilde{12})(45)$
$\mathbb{P}_{(1,3,4,4,12)}[24]$	$(22,6,4,4,0)$	14	(25)	$(25)(34)$
$\mathbb{P}_{(1,3,4,8,8)}[24]$	$(22,6,4,1,1)$	22	$(45)_{0,2}$	$(\widetilde{12})(45)_{0,2}$
$\mathbb{P}_{(1,3,6,6,8)}[24]$	$(22,6,2,2,1)$	18	$(34)_{0,2,4}$	$(34)_{0,2,4}$
$\mathbb{P}_{(2,3,3,4,12)}[24]$	$(10,6,6,4,0)$	14	(45)	$(23)(45)$
$\mathbb{P}_{(2,3,3,8,8)}[24]$	$(10,6,6,1,1)$	40	$(23)(45)_{0,\ldots,12;0,2}$	$(23)(45)_{0,\ldots,12;0,2}$
$\mathbb{P}_{(1,2,4,7,14)}[28]$	$(26,12,5,2,0)$	18	(45)	$(\widetilde{12})(45)$
$\mathbb{P}_{(1,1,3,10,15)}[30]$	$(28,28,8,1,0)$	12	(35)	$(12)(35)$
$\mathbb{P}_{(1,2,2,10,15)}[30]$	$(28,13,13,1,0)$	10	RS	(23)
$\mathbb{P}_{(1,2,6,6,15)}[30]$	$(28,13,3,3,0)$	20	$(34)_{0,2,4,6}$	$(34)_{0,2,4,6}$
$\mathbb{P}_{(1,3,5,6,15)}[30]$	$(28,8,4,3,0)$	16	(35)	$(\widetilde{12})(35)$
$\mathbb{P}_{(2,2,5,6,15)}[30]$	$(13,13,4,3,0)$	16	(35)	$(12)(35)$
$\mathbb{P}_{(2,3,5,5,15)}[30]$	$(13,8,4,4,0)$	16	(25)	$(34)(25)$
$\mathbb{P}_{(2,3,5,10,10)}[30]$	$(13,8,4,1,1)$	26	$(45)_{0,2}$	$(45)_{0,2}$
$\mathbb{P}_{(1,2,3,12,18)}[36]$	$(34,16,10,1,0)$	12	RS	RS
$\mathbb{P}_{(1,2,6,9,18)}[36]$	$(34,16,4,2,0)$	26	$(45)-(35)$	$(\widetilde{12})(45)-(\widetilde{12})(35)$
$\mathbb{P}_{(1,2,9,12,12)}[36]$	$(34,16,2,1,1)$	34	$(45)_{0,2}$	$(\widetilde{12})(45)_{0,2}$
$\mathbb{P}_{(1,4,4,9,18)}[36]$	$(34,7,7,2,0)$	40	$(23)(45)_{0,\ldots,14;0}$	$(23)(45)_{0,\ldots,14;0}$
$\mathbb{P}_{(2,3,4,9,18)}[36]$	$(16,10,7,2,0)$	22	(45)	$(\widetilde{13})(45)$
$\mathbb{P}_{(1,1,8,10,20)}[40]$	$(38,38,3,2,0)$	24	(45)	$(12)(45)$
$\mathbb{P}_{(1,4,5,10,20)}[40]$	$(38,8,6,2,0)$	26	$(45)-(25)$	$(\widetilde{13})(45)-(\widetilde{13})(25)$
$\mathbb{P}_{(2,5,5,8,20)}[40]$	$(18,6,6,3,0)$	16	RS	(23)
$\mathbb{P}_{(1,3,3,14,21)}[42]$	$(40,12,12,1,0)$	14	RS	(23)
$\mathbb{P}_{(1,6,7,7,21)}[42]$	$(40,5,4,4,0)$	18	RS	(34)
$\mathbb{P}_{(1,6,7,14,14)}[42]$	$(40,5,4,1,1)$	36	$(45)_{0,2}$	$(45)_{0,2}$
$\mathbb{P}_{(2,2,3,14,21)}[42]$	$(19,19,12,1,0)$	16	(35)	$(12)(35)$
$\mathbb{P}_{(2,6,6,7,21)}[42]$	$(19,5,5,4,0)$	36	$(23)(45)_{0,\ldots,10;0}$	$(23)(45)_{0,\ldots,10;0}$
$\mathbb{P}_{(1,5,9,15,15)}[45]$	$(43,7,3,1,1)$	40	$(45)_{0,2}$	$(45)_{0,2}$
$\mathbb{P}_{(1,1,6,16,24)}[48]$	$(46,46,6,1,0)$	20	(35)	$(12)(35)$
$\mathbb{P}_{(1,3,4,16,24)}[48]$	$(46,14,10,1,0)$	18	(25)	$(\widetilde{13})(25)$
$\mathbb{P}_{(1,3,8,12,24)}[48]$	$(46,14,4,2,0)$	34	$(45)-(35)$	$(\widetilde{12})(45)-(\widetilde{12})(35)$
$\mathbb{P}_{(2,3,3,16,24)}[48]$	$(22,14,14,1,0)$	16	RS	(23)
$\mathbb{P}_{(1,2,12,15,30)}[60]$	$(58,28,3,2,0)$	36	(45)	$(\widetilde{12})(45)$
$\mathbb{P}_{(1,3,6,20,30)}[60]$	$(58,18,8,1,0)$	22	(35)	(35)
$\mathbb{P}_{(1,4,10,15,30)}[60]$	$(58,13,4,2,0)$	42	$(45)-(35)$	$(\widetilde{12})(45)-(\widetilde{12})(35)$
$\mathbb{P}_{(1,4,15,20,20)}[60]$	$(58,13,2,1,1)$	54	$(45)_{0,2}$	$(45)_{0,2}$
$\mathbb{P}_{(1,5,12,12,30)}[60]$	$(58,10,3,3,0)$	52	$(34)(25)_{0,2,4,6;0}$	$(34)(25)_{0,2,4,6;0}$

Table 3: (*continued*)

Calabi-Yau	Gepner model	RR dim	RR vector space	RR charge lattice
$\mathbb{P}_{(1,12,12,15,20)}[60]$	$(58,3,3,2,1)$	48	$(23)_{0,2,4,6}$	$(23)_{0,2,4,6}$
$\mathbb{P}_{(2,3,5,20,30)}[60]$	$(28,18,10,1,0)$	22	(25)	$\widetilde{(13)}(25)$
$\mathbb{P}_{(2,3,10,15,30)}[60]$	$(28,18,4,2,0)$	42	$(45){-}(35)$	$\widetilde{(13)}(45){-}(35)$
$\mathbb{P}_{(3,5,10,12,30)}[60]$	$(18,10,4,3,0)$	28	(35)	(35)
$\mathbb{P}_{(4,5,6,15,30)}[60]$	$(13,10,8,2,0)$	38	$(45){-}(35)$	$(45){-}(35)$
$\mathbb{P}_{(2,3,6,22,33)}[66]$	$(31,20,9,1,0)$	20	RS	RS
$\mathbb{P}_{(1,10,10,14,35)}[70]$	$(68,5,5,3,0)$	48	$(23)_{0,\dots,10}$	$(23)_{0,\dots,10}$
$\mathbb{P}_{(2,5,14,14,35)}[70]$	$(33,12,3,3,0)$	56	$(34)(25)_{0,2,4,6;0}$	$(34)(25)_{0,2,4,6;0}$
$\mathbb{P}_{(1,2,9,24,36)}[72]$	$(70,34,6,1,0)$	30	(35)	$\widetilde{(12)}(35)$
$\mathbb{P}_{(1,8,9,18,36)}[72]$	$(70,7,6,2,0)$	40	(45)	(45)
$\mathbb{P}_{(1,6,6,26,39)}[78]$	$(76,11,11,1,0)$	48	$(23)_{0,\dots,22}$	$(23)_{0,\dots,22}$
$\mathbb{P}_{(1,1,12,28,42)}[84]$	$(82,82,5,1,0)$	24	RS	RS
$\mathbb{P}_{(1,6,7,28,42)}[84]$	$(82,12,10,1,0)$	32	(25)	$\widetilde{(13)}(25)$
$\mathbb{P}_{(1,6,14,21,42)}[84]$	$(82,12,4,2,0)$	62	$(45){-}(35){-}(25)$	$\widetilde{(12)}(45){-}\widetilde{(12)}(35){-}(25)$
$\mathbb{P}_{(2,7,12,21,42)}[84]$	$(40,10,5,2,0)$	48	(45)	(45)
$\mathbb{P}_{(2,12,21,21,28)}[84]$	$(40,5,2,2,1)$	72	$(34)_{0,2,4}$	$(34)_{0,2,4}$
$\mathbb{P}_{(3,4,14,21,42)}[84]$	$(26,19,4,2,0)$	58	$(45){-}(35)$	$(45){-}(35)$
$\mathbb{P}_{(1,5,9,30,45)}[90]$	$(88,16,8,1,0)$	36	(35)	$\widetilde{(12)}(35)$
$\mathbb{P}_{(2,10,15,18,45)}[90]$	$(43,7,4,3,0)$	40	(35)	(35)
$\mathbb{P}_{(1,3,12,32,48)}[96]$	$(94,30,6,1,0)$	38	(35)	(35)
$\mathbb{P}_{(1,10,22,22,55)}[110]$	$(108,9,3,3,0)$	80	$(34)_{0,2,4,6}$	$(34)_{0,2,4,6}$
$\mathbb{P}_{(1,4,15,40,60)}[120]$	$(118,28,6,1,0)$	50	(35)	$\widetilde{(12)}(35)$
$\mathbb{P}_{(1,5,24,30,60)}[120]$	$(118,22,3,2,0)$	68	(45)	(45)
$\mathbb{P}_{(1,15,20,24,60)}[120]$	$(118,6,4,3,0)$	68	$(35){-}(25)$	$(35){-}(25)$
$\mathbb{P}_{(1,15,24,40,40)}[120]$	$(118,6,3,1,1)$	112	$(45)_{0,2}$	$(45)_{0,2}$
$\mathbb{P}_{(2,3,15,40,60)}[120]$	$(58,38,6,1,0)$	48	(35)	(35)
$\mathbb{P}_{(3,5,12,40,60)}[120]$	$(38,22,8,1,0)$	46	(35)	(35)
$\mathbb{P}_{(1,2,18,42,63)}[126]$	$(124,61,5,1,0)$	36	RS	RS
$\mathbb{P}_{(1,14,20,35,70)}[140]$	$(138,8,5,2,0)$	90	$(45){-}(25)$	$(45){-}(25)$
$\mathbb{P}_{(2,5,28,35,70)}[140]$	$(68,26,3,2,0)$	80	(45)	(45)
$\mathbb{P}_{(1,12,13,52,78)}[156]$	$(154,11,10,1,0)$	48	RS	RS
$\mathbb{P}_{(1,12,26,39,78)}[156]$	$(154,11,4,2,0)$	108	$(45){-}(35)$	$(45){-}(35)$
$\mathbb{P}_{(1,12,39,52,52)}[156]$	$(154,11,2,1,1)$	144	$(45)_{0,2}$	$(45)_{0,2}$
$\mathbb{P}_{(1,3,24,56,84)}[168]$	$(166,54,5,1,0)$	48	RS	RS
$\mathbb{P}_{(1,6,21,56,84)}[168]$	$(166,26,6,1,0)$	70	$(35){-}(25)$	$(35){-}(25)$
$\mathbb{P}_{(3,4,21,56,84)}[168]$	$(54,40,6,1,0)$	68	(35)	(35)
$\mathbb{P}_{(2,3,30,70,105)}[210]$	$(103,68,5,1,0)$	60	RS	RS
$\mathbb{P}_{(1,10,44,55,110)}[220]$	$(218,20,3,2,0)$	132	$(45){-}(25)$	$(45){-}(25)$
$\mathbb{P}_{(1,15,24,80,120)}[240]$	$(238,14,8,1,0)$	102	$(35){-}(25)$	$(35){-}(25)$
$\mathbb{P}_{(1,12,39,104,156)}[312]$	$(310,24,6,1,0)$	134	$(35){-}(25)$	$(35){-}(25)$
$\mathbb{P}_{(1,7,48,112,168)}[336]$	$(334,46,5,1,0)$	96	RS	RS

Table 3: (*continued*)

Calabi-Yau	Gepner model	RR dim	RR vector space	RR charge lattice
$\mathbb{P}_{(2,7,54,126,189)}[378]$	$(187, 52, 5, 1, 0)$	108	RS	RS
$\mathbb{P}_{(1,20,84,105,210)}[420]$	$(418, 19, 3, 2, 0)$	240	(45)	(45)
$\mathbb{P}_{(3,7,60,140,210)}[420]$	$(138, 58, 5, 1, 0)$	120	RS	RS
$\mathbb{P}_{(2,33,42,154,231)}[462]$	$(229, 12, 9, 1, 0)$	160	(25)	(25)
$\mathbb{P}_{(1,24,75,200,300)}[600]$	$(598, 23, 6, 1, 0)$	240	(35)	(35)
$\mathbb{P}_{(1,21,132,308,462)}[924]$	$(922, 42, 5, 1, 0)$	276	(25)	(25)
$\mathbb{P}_{(2,21,138,322,483)}[966]$	$(481, 44, 5, 1, 0)$	288	(25)	(25)
$\mathbb{P}_{(1,42,258,602,903)}[1806]$	$(1804, 41, 5, 1, 0)$	504	RS	RS

Table 3: The models where the RR vector space is generated by tensor product and conventional permutation branes.

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